Quantum Mechanics
(II)

Physics

Sol^n of the Schroedinger Eq^n :-

The time-dependent Schroedinger eq^n is given by :- $H_0 \psi_n = E_n \psi_n$

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\psi = i\hbar\frac{\delta\psi}{\delta t} \quad \text{—— (1)}$$

The eq^n is a partial differential eq^n in four variables i.e. the three position coordinates of the particle x, y, z and the time t.

In the case where the potential energy V is independent of time and depends only on position, then according to Schroedinger this wave is standing wave.
In such others the phase of vibration is same everywhere : while in progressive waves, at any moment, the progression of phase along the wave-train exists.

If ψ represents standing wave, time must occur as a separate factor i.e, if V is independent of time, the position and time coordinate are separate in eqⁿ ①, so that Ψ may expressed in the form

$$\Psi = \underbrace{\phi(x,y,z)}_{\text{indep. of time}} \underbrace{u(t)}_{\text{indep. of position}} \quad \text{—②}$$

Put ② in ①, we get

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(x,y,z)\right]\phi(x,y,z)$$

$$= i\hbar\frac{\delta}{\delta t}\left\{\phi(x,y,z)u(t)\right\}$$

$\div$ by $\phi(x,y,z)u(t)$

$$\frac{1}{\phi(x,y,z)}\left[-\frac{\hbar^2}{2m}\nabla^2 + V(x,y,z)\right]\phi(x,y,z)$$

$$= i\hbar\frac{1}{u(t)}\frac{\delta}{\delta t}u(t) \quad \text{—③}$$

Let

$$\frac{1}{\phi(x,y,z)}\left[-\frac{\hbar^2}{2m}\nabla^2 + V(x,y,z)\right]\phi(x,y,z) = E$$

$$\nabla^2\phi + \frac{2m}{\hbar^2}(E-V)\phi = 0 \quad \text{—④}$$

$$i\hbar \cdot \frac{1}{u(t)} \frac{\delta u(t)}{\delta t} = E \quad \text{—— (5)}$$

The sol^n of eq^n (5) may be written as

$$u(t) = e^{Et/i\hbar} = e^{-iEt/\hbar} \quad \text{—— (6)}$$

$$\boxed{\Psi(t) = \phi(x,y,z)\, e^{-iEt/\hbar}} \quad (7)$$

Most general sol^n Ψ of Schroedinger eq^n for time-independent potential energy can be written as

$$\Psi = a_1 \phi_1(x,y,z)\, e^{-iE_1 t/\hbar} + a_1 \phi_2(x,y,z) e^{-iE}$$

$$+ \ldots + a_n \phi_n(x,y,z)\, e^{-iE_n t/\hbar} + \ldots$$

$$\Psi = \sum_{n=1}^{\infty} a_n \phi_n(x,y,z)\, e^{-iE_n t/\hbar} \quad \text{—— (8)}$$

This theory is useful for studying process of absorption and emission of radiation by atoms or more generally

Time Dependent Perturbation Theory :-

This theory was given by Dirac in 1926.
When the hamiltonian of a system depends upon time there are no stationary states of schroedinger eqn. In this position it is almost impossible to obtain Exact solutions of the Schroedinger eqn when Hamiltonian depends upon time. Therefore, such types of eqn's are solved by some approximate methods and one of the approximation method is time dependent perturbation theory or Method of variation of constant.

e.g. ⇒ 1. Transition of particle from ground state to higher excited state.
2. Effect of electric field or magnetic field which are time dependent.
3. Spin-orbit interaction etc.

for treating transitions of the quantum mechanical systems from one energy level to another (also the ionization of H atom) (5)

Schroedinger Time independent eq^n is given by :-

$$H_0 \Psi_n = E_n \Psi_n$$

Its stationary state sol^n is given by :-

$$\Psi_n(t) = \phi_n(r)\, e^{-iE_n t/\hbar}$$

here, ϕ_n is the function of position only not time.

But if the Hamiltonian is time-dependent i.e; Schroedinger eq^n is time dependent

$$i\hbar \frac{\delta \Psi}{\delta t} = H\Psi \quad \text{--- (1)}$$

then its stationary state sol^n is impossible bz energy of system is no longer constant

For this Hamiltonian is written as i.e in such case Hamiltonian is supposed to be formed of two parts

$$H = H_0 + H'(t) \quad \text{--- (2)}$$

H_0: unperturbated Hamiltonian or Exact value of Hamiltonian

$H'(t)$: Perturbated Hamiltonian which depends upon time 't'

A direct solution of eqn (1) can not be obtained we deal the problem in different manner. At a particular instant assumed initial

i.e, H will also depends upon time.

Now, let us consider a new normalized eigen function ϕ_n and its eigen value is E_n. Therefore,

$$H_0 \phi_n = E_n \phi_n \quad \text{--- (3)}$$

H' and H_0 are operators. H' is very small in comparison to H_0 but they may be comparable to each other. It is one of the possibility.

H_0 and H' both are Hermitian operators. They always give real value which will not be negative.

Let the soln of this time dependent schroedinger eqn is given by:-

$$\psi(t) = \sum a_n(t)\, \phi_n(r)\, e^{(-iE_n t/\hbar)} \quad \text{--- (4)}$$

where a_n is time dependent or unitary operator
time dependent